科技部補助專題研究計畫成果報告
期末報告

非阿基米德動態系統與枝群

計 畫 類 別 ： 個別型計畫
計 畫 編 號 ： MOST 106-2115-M-003-001-
執 行 期 間 ： 106年08月01日至107年07月31日
執 行 單 位 ： 國立臺灣師範大學數學系（所）

計畫主持人： 李華介

計畫參與人員： 碩士班研究生-兼任助理：莊智宇

中 華 民 國 107 年 10 月 27 日

中 文 摘 要 ： 群論中的樹狀表現與非阿基米德動態系統中伽羅瓦群對其根的作用有相同的概念。本計畫中我們探討樹狀表現理論中的枝群與前述的伽羅瓦群的表現理論之相關性。從此計畫中我們了解到一般樹狀表現對非阿基米德動態系統的伽羅瓦群的表現理論來說太大而難以具體描繪。而枝群對描繪非阿基米德動態系統相關性非常的高，尤其對其伽羅瓦群的裂解群的描繪非常類似。最後我們附上在裂解群發表的研究成果。

中 文 關 鍵 詞 ： 樹狀表現，非阿基米德動態系統，枝群，裂解群。

英 文 摘 要 ： The action of the Galois groups on the roots of iterates of a noninvertible power series can be regarded as the automorphism groups of locally finite trees. In this project, we study both the branch groups and the Galois groups of the extension of the roots of iterates. From our study, we realize that the usual tree representation from the point of view of wreath product is too large for us to handle the iterated Galois groups. Branch groups on the other hand seem perfectly related to the action of the ramification groups on the roots of the iterated power series. We also include the results of my study about the ramification groups in this report.

英 文 關 鍵 詞 ： Tree representation, Nonarchimedean dynamical systems, Branch groups, Ramification groups.

1 前言

The action of the Galois groups on the roots of iterates of a noninvertible power series can be regarded as the automorphism groups of locally finite trees. In this project, we study both the branch groups and the Galois groups of the extension of the roots of iterates. From our study, we realize that the usual tree representation from the point of view of wreath product is too large for us to handle the iterated Galois groups. Branch groups on the other hand seem perfectly related to the action of the ramification groups on the roots of the iterated power series. We also include the results of my study about the ramification groups in this report.

2 研究目的

Let K be a finite extension of $\mathbb{Q}_p$ and let $\mathscr{O}$ be its ring of integers with residue field k. We call a power series $g(x) \in \mathscr{O}[[x]]$ stable if $g(0) = 0$ and $g'(0)$ is neither 0 nor a root of 1. As usual, we write $h(g(x)) = h \circ g(x)$; in a less standard notation, we denote by $g^{\circ n}(x)$ the n-fold composition of $g(x)$ with itself; this make sense for negative n in case $g(x)$ is invertible. Let $\overline{K}$ be the algebraic closure of K with ring of integers $\overline{\mathscr{O}}$ and maximal ideal $\overline{\mathscr{M}}$. An element $\alpha \in \overline{\mathscr{M}}$ satisfying $f^{\circ n}(\alpha) = 0$ for some $n \in \mathbb{N}$ is called a root of iterates of $f(x)$.

Let $f(x) \in \mathscr{O}[[x]]$ be a non-invertible series with only simple roots of iterates. Theses roots of iterates form a spherically homogeneous tree T with the distinguished vertex (root vertex) 0. The branch degree of the tree at level 1 is $d-1$ and at level n is d for $n > 1$, where d is the Weierstrass degree of $f(x)$. Let G be the Galois group of the extension over K jointed by all the roots of iterates of $f(x)$. Then G can naturally be considered as a subgroup of $\mathrm{Aut}(T)$ (the automorphism group of T). Boston and Jones have similar consideration and call it "arboreal Galois representation" ([4]).

In the case of pro-p-extensions unramified at p, there is the fundamental conjectures of Fontaine-Mazur [10] claiming that any p-adic representations should have finite image. In other words, p-adic representations say little about such Galois groups. At present, two types of just-infinite pro-p groups has been considered for possible Galois representation with large image: certain subgroups of the Nottingham groups $\mathscr{N}(k)$ (automorphisms $\sigma \in \mathrm{Aut}_k(k((x)))$ with $\sigma(x) \equiv x \pmod{x^2}$) [7] and Grigorchuk's Branch groups (certain automorphism groups of rooted trees) [11]. The field of norms construction yields representations of local Galois groups into $\mathcal{N}(k)$ by Fesenko [8], and Boston [3] conjectures that branch groups might provide representations with large image (measured by Hausdorff dimension).

For a non-invertible series $f(x)$, the set U of invertible series which commutes with $f(x)$ forms a group under composition. The action of U on the roots of iterates of $f(x)$ also can be regarded as a subgroup of $\mathrm{Aut}(T)$. Moreover elements of U also commutes with elements of G when both of them are considered as elements in $\mathrm{Aut}(T)$. In other words, U is contained in the center of G. Lubin's conjecture says that if U is not trivial, then $f(x)$ must be related to a formal group [19]. This is true when $f(x)$ is an endomorphism of a formal group. In particular, when $f(x)$ is an endomorphism of Lubin-Tate formal group, G is contained in U when both of U, G are considered as subgroups of $\mathrm{Aut}(T)$ [20].

In this project, we wish to find a suitable branch group so that G can be considered as a subgroup of it. This might allow us to investigate the structure of G, something unapproachable by those method we used before. We will focus on Lubin's conjecture. Hopefully, some knowledge of branch group can help us to understand the size of the center of G.

The theory of branch group was developed during last 20 years. Most of the developing was in connection with the theory of groups generated by finite automata [5, 6], problem of Burnside type [12, 13, 15] and problems on growth in groups [12, 13, 14]. The second goal of this project is hoping some methods from non-archimedean dynamical system can be applied to help developing the theory of branch group.

3 文獻探討

In this report, all the trees have the property that every vertex of the same level has the same degree (也就是說同一層的點都有相同個數的分枝). 在探討這樣的 trees 的 automorphisms group, 我們先將每一個 vertex 標好號, 以方便用 permutations 來表示其 automorphisms. 我目前知道有以下兩種方式來探討其 automorphisms group. 我分別用 A, B 來表示. 其中 A 可參考 [16], B 可參考 [23, Chapter 7].

A: 假設 T_n 是 n-level 的 tree, 我們可以將 T_n 看成是 a copy of T_1 (和 T_n 同樣 root 的 1-level subtree) 然後每個 T_1 上的 leaf (tree 的 leaf 為其最上一層的 vertex) 再加上 $n-1$ 層的 tree T'_{n-1}. 所以 T_n 我們可以看成是 $T'_{n-1} \times T_1$,

每個 T_n 的 leaf 我們用 (x,i) 來表示, 其中 i 是 T_1 上的 leaf, x 是 T'_{n-1} 上的 leaf. 所以 $\mathrm{Aut}(T_n)$ 上的元素 σ, 我們就可以用 $\sigma = ((a_1,\ldots,a_m),b)$ 來表示, 其中 m 表示 T_1 上 leaves 的個數, 而 $b \in \mathrm{Aut}(T_1)$, $a_1,\ldots,a_m \in \mathrm{Aut}(T'_{n-1})$. σ 對 T_n 的作用就是先將 m copies of T'_{n-1} 分別用 $a_1,\ldots,a_m$ 作用, 然後再用 b 將這 m 個 T'_{n-1} 移動. 具體來說對於 $(x,i) \in T_n$, 我們有 [1, Page 5]

$$\sigma(x,i) = ((a_1,\ldots,a_m),b)(x,i) = (a_i(x),b(i)).$$

B: 假設 T_n 是 n-level 的 tree, 我們可以將 T_n 看成是 a copy of T_{n-1} (和 T_n 同樣 root 的 $n-1$-level subtree) 然後每個 T_{n-1} 上的 leaf 再加上 1 層的 tree T'_1. 所以 T_n 我們可以看成是 $T'_1 \times T_{n-1}$, 每個 T_n 的 leaf 我們用 (y,j) 來表示, 其中 j 是 T_{n-1} 上的 leaf, y 是 T'_1 上的 leaf. 所以 $\mathrm{Aut}(T_n)$ 上的元素 σ, 我們就可以用 $\sigma = ((c_1,\ldots,c_M),d)$ 來表示, 其中 M 表示 T_{n-1} 上 leaves 的個數, 而 $d \in \mathrm{Aut}(T_{n-1})$, $c_1,\ldots,c_M \in \mathrm{Aut}(T'_1)$. σ 對 T_n 的作用就是先用 d 將這 M 個 T_{n-1} 移動再對 M copies of T'_1 分別用 $c_1,\ldots,c_M$ 作用. 具體來說對於 $(x,i) \in T_n$, 我們有 [23, Page 172–Page 173]

$$\sigma(y,j) = ((c_1,\ldots,c_M),d)(y,j) = (c_{d(j)}(y),d(j)).$$

接下來我們探討如何由 A, B 這兩種看法, 利用 wreath product 來描繪 $\mathrm{Aut}(T_n)$. 在此之前我們來看兩種 group actions. 在這個部分, 我們暫時用以下的 notations. 假設 G 是一個 group 考慮 $G^n = \underbrace{G \times \cdots \times G}_{n}$ 以及 $\sigma \in S_n$ 對 G^n 的 action. 若 $\vec{\rho} \in G^n$, 我們用上標 $\vec{\rho} = (\rho^1,\ldots,\rho^i,\ldots,\rho^n) \in G^n$ 來表示 G^n 的元素. 也就是說 $\vec{\rho}$ 的 i-th entry 為 $\rho^i \in G$. 當 $\vec{\rho}$ 經由 index 的 permutation 變成 G^n 另一個元素時, 我們用 ρ_i^j 表示其 i-th entry 是原來 $\vec{\rho}$ 的 j-th entry ρ^j. 現若 $\sigma \in S_n$, 我們考慮 right action

$$\vec{\rho} * \sigma = (\ldots,\rho_i^{\sigma(i)},\ldots),$$

也就是說 $\vec{\rho} * \sigma$ 的 i-th entry 就是原來 $\vec{\rho}$ 的 $\sigma(i)$-th entry (或是說 $\vec{\rho}$ 的 i-th entry 會是新的 $\vec{\rho} * \sigma$ 的 $\sigma^{-1}(i)$-th entry). 我們也考慮 left action

$$\sigma * \vec{\rho} = (\ldots,\rho^i_{\sigma(i)},\ldots),$$

也就是說原來 $\vec{\rho}$ 的 i-th entry 會是新的 $\sigma * \vec{\rho}$ 的 $\sigma(i)$-th entry (或是說 $\sigma * \vec{\rho}$ 的 i-th entry 就是 $\vec{\rho}$ 的 $\sigma^{-1}(i)$-th entry).

為了讓大家熟悉這個 notation, 我們檢查它們是否為 S_n 對 G^n 的 group operations. 首先看 right action, 假設 $\sigma,\tau \in S_n$, 我們有 $\vec{\rho} * (\tau\sigma) =$

$(\ldots,\rho_i^{(\tau\sigma)(i)},\ldots)$, 表示 $\vec{\rho}*(\tau\sigma)$ 的 i-th entry 就是原來 $\vec{\rho}$ 的 $(\tau\sigma)(i)$-th entry. 另一方面我們檢查 $(\vec{\rho}*\tau)*\sigma$ 的 i-th entry. 依定義 $(\vec{\rho}*\tau)*\sigma$ 的 i-th entry 就是 $\vec{\rho}*\tau$ 的 $\sigma(i)$-th entry, 而 $\vec{\rho}*\tau=(\cdots,\rho_i^{\tau(i)},\ldots)$, 表示其 $\sigma(i)$-th entry 就是 $\vec{\rho}$ 的 $\tau(\sigma(i))$-th entry (即 $(\tau\sigma)(i)$-th entry, $\rho^{(\tau\sigma)(i)}$). 因為這對每個 i 皆成立, 故得 $\vec{\rho}*(\tau\sigma)=(\vec{\rho}*\tau)*\sigma$. 考慮 G^n 上 G 的 direct product 的 group structure, 我們也可檢查這個 right action, 對任意 $\sigma\in S_n$ 給了一個 G^n 的 endomorphism (因此是 automorphism). 假設 $\vec{\rho}=(\ldots,\rho^i,\ldots),\vec{\gamma}=(\ldots,\gamma^i,\ldots)$ 以及 $\sigma\in S_n$, 則 $(\vec{\rho}\vec{\gamma})*\sigma$ 的 i-th entry 是原先 $\vec{\rho}\vec{\gamma}$ 的 $\sigma(i)$-th entry. 而 $\vec{\rho}\vec{\gamma}=(\ldots,\rho^i\gamma^i,\ldots)$, 也就是說 $\vec{\rho}\vec{\gamma}$ 的 $\sigma(i)$-th entry 就是 $\rho^{\sigma(i)}\gamma^{\sigma(i)}$. 另一方面

$$(\vec{\rho}*\sigma)(\vec{\gamma}*\sigma)=(\ldots,\rho_i^{\sigma(i)},\ldots)(\ldots,\gamma_i^{\sigma(i)},\ldots)=(\ldots,\rho_i^{\sigma(i)}\gamma_i^{\sigma(i)},\ldots).$$

亦即其 i-th entry 亦為 $\rho^{\sigma(i)}\gamma^{\sigma(i)}$, 因此得證 $(\vec{\rho}\vec{\gamma})*\sigma=(\vec{\rho}*\sigma)(\vec{\gamma}*\sigma)$.

接著看 left action, 假設 $\sigma,\tau\in S_n$, 我們有 $(\tau\sigma)*\vec{\rho}=(\ldots,\rho^i_{\tau\sigma(i)},\ldots)$, 表示 $(\tau\sigma)*\vec{\rho}$ 的 $\tau\sigma(i)$-th entry 就是原來的 ρ^i. 另一方面我們檢查 $\tau*(\sigma*\vec{\rho})$ 的 $\tau\sigma(i)$-th entry. 依定義 $\tau*(\sigma*\vec{\rho})$ 的 $\tau\sigma(i)$-th entry 就是 $\sigma*\vec{\rho}$ 的 $\sigma(i)$-th entry, 而 $\sigma*\vec{\rho}=(\cdots,\rho^i_{\sigma(i)},\ldots)$, 表示其 $\sigma(i)$-th entry 就是 ρ^i. 因此知 $(\tau\sigma)*\vec{\rho}=\tau*(\sigma*\vec{\rho})$. 和 right action 一樣的, 我們可以證得 $\sigma*(\vec{\rho}\vec{\gamma})=(\sigma*\vec{\rho})(\sigma*\vec{\gamma})$.

我們回到探討如何由 A, B 這兩種看法, 利用 wreath product 來描繪 $\mathrm{Aut}(T_n)$, 我們回到原來的 notations.

A: 假設 $\sigma=((a_1,\ldots,a_m),b)$, $\tau=((a'_1,\ldots,a'_m),b')$ 且 (x,i) 是 T_n 的一個 leaf, 其中 $a_i,a'_i\in\mathrm{Aut}(T'_{n-1})$, $b,b'\in\mathrm{Aut}(T_1)$ 以及 x 是 T'_{n-1} 的 leaf, i 是 T_1 的 leaf. 假設 $\tau\sigma=((a''_1,\ldots,a''_m),b'')$, 則依定義 $(\tau\sigma)(x,i)=(a''_i(x),b''(i))$. 另一方面

$$(\tau\sigma)(x,i)=\tau(\sigma(x,i))=\tau(a_i(x),b(i))=(a'_{b(i)}(a_i(x)),b'(b(i))).$$

因此知 $a''_i(x)=a'_{b(i)}(a_i(x))=(a'_{b(i)}a_i)(x)$ 以及 $b''(i)=b'(b(i))=(b'b)(i)$. 由於這對所有的 i, x 皆成立, 因此得 $a''_i=a'_{b(i)}a_i$ 以及 $b''=b'b$. 若令 $\vec{a}=(a_1,\ldots,a_m)$, $\vec{a}'=(a'_1,\ldots,a'_m)$ 以及 $\vec{a}''=(a''_1,\ldots,a''_m)$. 上式告訴我們 $\tau\sigma=(\vec{a}',b')\cdot(\vec{a},b)=(\vec{a}'',b'b)$ 其中 $\vec{a}''$ 的 i-th entry 為 $a'_{b(i)}a_i$. 然而 $a'_{b(i)}$ 恰為前面介紹的 right action $\vec{a}'*b$ 的 i-th entry, 而 $\vec{a}$ 的 i-th entry 為 a_i, 故 $(\vec{a}'*b)\vec{a}$ 的 i-th entry 亦為 $a'_{b(i)}a_i$. 所以我們有 $\vec{a}''=(\vec{a}'*b)\vec{a}$. 也因此有以下的式子

$$(\vec{a}',b')\cdot(\vec{a},b)=((\vec{a}'*b)\vec{a},b'b). \tag{3.1}$$

注意這裡 $(\vec{a}'*b)\vec{a}$ 是 $\mathrm{Aut}(T'_{n-1})^m = \underbrace{\mathrm{Aut}(T'_{n-1})\times\cdots\times\mathrm{Aut}(T'_{n-1})}_{m}$ 上的運算, 而 $b'b$ 是 $\mathrm{Aut}(T_1)$ 的運算. 我們可以驗證式子 (1) 定出了一個 group structure, 也就是 a semidirect product of $\mathrm{Aut}(T'_{n-1})$ by $\mathrm{Aut}(T_1)$(即 $\mathrm{Aut}(T'_{n-1})\rtimes\mathrm{Aut}(T_1)$). 這裡我們僅驗證結合律. Consider $\vec{a},\vec{a}',\vec{a}''\in\mathrm{Aut}(T'_{n-1})$ and $b,b',b''\in\mathrm{Aut}(T_1)$. Then

$$(\vec{a}'',b'')\big((\vec{a}',b')(\vec{a},b)\big) = (\vec{a}'',b'')((\vec{a}'*b)\vec{a},b'b) = ((\vec{a}''*(b'b))(\vec{a}'*b)\vec{a},b''b'b).$$

On the other hand,

$$\big((\vec{a}'',b'')(\vec{a}',b')\big)(\vec{a},b) = ((\vec{a}''*b')\vec{a}',b''b')(\vec{a},b) = \Big(\Big(\big((\vec{a}''*b')\vec{a}'\big)*b\Big)\vec{a},b''b'b\Big).$$

Since right action induces a group homomorphism, we have

$$\big((\vec{a}''*b')\vec{a}'\big)*b = \big((\vec{a}''*b')*b\big)(\vec{a}'*b) = (\vec{a}''*(b'b))(\vec{a}'*b).$$

Therefore,

$$(\vec{a}'',b'')\big((\vec{a}',b')(\vec{a},b)\big) = \big((\vec{a}'',b'')(\vec{a}',b')\big)(\vec{a},b).$$

也因此我們可推得 $\mathrm{Aut}(T_n)\simeq\mathrm{Aut}(T'_{n-1})\wr_\Lambda\mathrm{Aut}(T_1)$, 其中 $\Lambda=\{1,\ldots,m\}$ 且 m 是 T_n 中 level 1 的 vertices 的個數.

B: 假設 $\sigma=((c_1,\ldots,c_M),d)$, $\tau=((c'_1,\ldots,c'_M),d')$ 且 (y,j) 是 T_n 的一個 leaf, 其中 $c_i,c'_i\in\mathrm{Aut}(T'_1)$, $b,b'\in\mathrm{Aut}(T_{n-1})$ 以及 y 是 T'_1 的 leaf, j 是 T_{n-1} 的 leaf. 假設 $\tau\sigma=((c''_1,\ldots,c''_M),d'')$, 則依定義 $(\tau\sigma)(y,j)=(c''_{d''(j)}(y),d''(j))$. 另一方面

$$(\tau\sigma)(y,j)=\tau(\sigma(y,j))=\tau(c_{d(j)}(y),d(j))=(c'_{d'(d(j))}(c_{d(j)}(y)),d'(d(j))).$$

因此知 $c''_{d''(j)}(y)=c'_{d'(d(j))}(c_{d(j)}(y))=(c'_{d'(d(j))}c_{d(j)})(y)$ 以及 $d''(j)=d'(d(j))=(d'd)(j)$. 由於這對所有的 j, y 皆成立, 因此得 $c''_{d''(j)}=c'_{d'(d(j))}c_{d(j)}$ 以及 $d''=d'd$. 若令 $\vec{c}=(c_1,\ldots,c_M)$, $\vec{c}'=(c'_1,\ldots,c'_M)$ 以及 $\vec{c}''=(c''_1,\ldots,c''_M)$. 上式告訴我們 $\tau\sigma=(\vec{c}',d')\cdot(\vec{c},d)=(\vec{c}'',d'd)$ 其中 $\vec{c}''$ 的 $(d'd)(j)$-th entry 為 $c'_{(d'd)(j)}c_{d(j)}$. 然而 $c'_{(d'd)(j)}$ 是 $\vec{c}'$ 的 $(d'd)(j)$-th entry, 而 $c_{d(j)}$ 為 $\vec{c}$ 的 $d(j)$-th entry 也恰為前面介紹的 left action $d'*\vec{c}$ 的 $(d'd)(j)$-th entry, 故 $\vec{c}'(d'*\vec{c})$ 的 $(d'd)(j)$-th entry 亦為 $c'_{(d'd)(j)}c_{d(j)}$. 因為這對所有的 j 成立, 所以我們有 $\vec{c}''=\vec{c}'(d'*\vec{c})$. 也因此有以下的式子

$$(\vec{c}',d')\cdot(\vec{c},d)=(\vec{c}'(d'*\vec{c}),d'd). \tag{3.2}$$

注意這裡 $\vec{c}'(d'*\vec{c})$ 是 $\mathrm{Aut}(T_1')^M = \underbrace{\mathrm{Aut}(T_1')\times\cdots\times\mathrm{Aut}(T_1')}_{M}$ 上的運算, 而 $d'd$ 是 $\mathrm{Aut}(T_{n-1})$ 的運算. 我們可以驗證式子 (2) 定出了一個 group structure (和 [23, Page 170] 的定法相同), 也就是 a semidirect product of $\mathrm{Aut}(T_1')$ by $\mathrm{Aut}(T_{n-1})$(即 $\mathrm{Aut}(T_1')\rtimes\mathrm{Aut}(T_{n-1})$). 這裡我們僅驗證結合律. Consider $\vec{c},\vec{c}',\vec{c}''\in\mathrm{Aut}(T_1')$ and $d,d',d''\in\mathrm{Aut}(T_{n-1})$. Then

$$(\vec{c}'',d'')\big((\vec{c}',d')(\vec{c},d)\big) = (\vec{c}'',d'')(\vec{c}'(d'*\vec{c}),d'd) = \big(\vec{c}''\big(d''*(\vec{c}'(d'*\vec{c})\big),d''d'd\big).$$

On the other hand,

$$\big((\vec{c}'',d'')(\vec{c}',d')\big)(\vec{c},d) = (\vec{c}''(d''*\vec{c}'),d''d')(\vec{c},d) = \big(\big(\vec{c}''(d''*\vec{c}')\big)((d''d')*\vec{c}),d''d'd\big).$$

Since left action induces a group homomorphism, we have

$$d''*(\vec{c}'(d'*\vec{c})) = (d''*\vec{c}')(d''*(d'*\vec{c})) = \big((d''*\vec{c}')\big)((d''d')*\vec{c}).$$

Therefore,

$$(\vec{c}'',d'')\big((\vec{c}',d')(\vec{c},d)\big) = \big((\vec{c}'',d'')(\vec{c}',d')\big)(\vec{c},d).$$

也因此我們可推得 $\mathrm{Aut}(T_n)\simeq\mathrm{Aut}(T_1')\wr_\Gamma\mathrm{Aut}(T_{n-1})$, 其中 $\Gamma=\{1,\ldots,M\}$ 且 M 是 T_n 中 level $n-1$ 的 vertices 的個數.

以上就是有關 tree automorphisms 的兩種看法. 我們探討 composites of polynomials 的 Galois group, 好像看法 B 比較直觀. Odoni [22] 就是用這種看法. 不過前面提過 [1] 是用 A 的看法. 最後我們利用 [22, Lemma 1.1] 來當一個例子.

Example 1. Let K be any field of characteristic 0, let $k\geq 2$ and let $t\in K$. Suppose that K contains all k-th roots of 1. Let $f(x)\in K[x]$ be a nonzero polynomial such that $f(x^k+t)$ is separable over K. If $\mathrm{Gal}(f(x)/K)=H$, then there is a canonical injection of $\mathrm{Gal}(f(x^k+t)/K)$ into $C_k\wr_\Lambda H$, where C_k is cyclic of order k and Λ is the set of roots of $f(x)$.

Proof. Let L be the splitting field of $f(x)$ and let F be the splitting field of $f(x^k+t)$, so we have $H=\mathrm{Gal}(f(x)/K)=\mathrm{Gal}(L/K)$ and $G=\mathrm{Gal}(f(x^k+t)/K)=\mathrm{Gal}(F/K)$. Fix ζ, a primitive k-th root of 1 and fix an order for $\Lambda=\{\lambda_1,\ldots,\lambda_M\}$. For $\lambda_j\in\Lambda$, we also fix a $\beta_{j,0}$ being a root of $x^k=\lambda_j-t$ and set $\beta_{j,l}=\zeta^l\beta_{j,0}$ for $1\leq l\leq k-1$.

For $\sigma\in G$, let $\sigma|_L=d$ and if $\sigma(\lambda_j)=\lambda_s$, we denote it by $d(j)=s$. Moreover, if $\sigma(\beta_{j,0})=\zeta^c\beta_{s,0}$, then we denote c for the $s=d(j)$-th entry of $\vec{c}$.

In this way, (i.e. sending σ to $(\vec{c}, d)$) we defined a function Φ from G into $C_k \wr_\Lambda H$. Clearly Φ is an one-to-one function. We need to check that Φ is a group homomorphism.

Suppose that $\sigma, \tau \in G$ and $\Phi(\sigma) = (\vec{c}, d)$, $\Phi(\tau) = (\vec{c}', d') \in C_k \wr_\Lambda H$, where $\vec{c} = (c_1, \ldots, c_M)$ and $\vec{c}' = (c'_1, \ldots, c'_M)$. In other words, for $\lambda_j \in \Lambda$ we have $\sigma(\lambda_j) = \lambda_{d(j)}$, $\tau(\lambda_j) = \lambda_{d'(j)}$ and $\sigma(\beta_{j,0}) = \zeta^{c_{d(j)}} \beta_{d(j),0}$, $\tau(\beta_{j,0}) = \zeta^{c'_{d'(j)}} \beta_{d'(j),0}$. We claim

$$\Phi(\tau\sigma) = (\vec{c}', d')(\vec{c}, d) = (\vec{c}' + (d' * \vec{c}), d'd)$$

(we write the group C_k additively). Clearly, we have

$$(\tau\sigma)(\lambda_j) = \tau(\sigma(\lambda_j)) = \tau(\lambda_{d(j)}) = \lambda_{d'(d(j))},$$

and hence $\tau\sigma|_L = d'd$. Now consider

$$(\tau\sigma)(\beta_{j,0}) = \tau(\sigma(\beta_{j,0})) = \tau(\zeta^{c_{d(j)}} \beta_{d(j),0}) = \zeta^{c_{d(j)}} \tau(\beta_{d(j),0}) = \zeta^{c_{d(j)}} \zeta^{c'_{d'(d(j))}} \beta_{d'(d(j)),0}.$$

In other words, if $\Phi(\tau\sigma) = (\vec{c}'', d'd)$, then the $d'(d(j))$-th entry of $\vec{c}''$ is $c'_{d'(d(j))} + c_{d(j)}$. $c'_{d'(d(j))}$ is by definition the $d'(d(j))$-th entry of $\vec{c}'$ and $c_{d(j)}$ is the $d(j)$-th entry of $\vec{c}$ and hence the $d'(d(j))$-th entry of $d' * \vec{c}$ by definition. Therefore, $c'_{d'(d(j))} + c_{d(j)}$ is the $d'(d(j))$-th entry of $\vec{c}' + (d' * \vec{c})$. Since this is true for every j, we get $\vec{c}'' = \vec{c}' + (d' * \vec{c})$ and complete the proof. □

4 研究方法

過去對於 Galois representation 皆著重於所謂 matrix representation. 事實上 Tree representation 從 Fontaione-Mazur conjecture [10] 的角度來說卻也相當重要，因為這個 conjecture 認為對於一些特殊的 Galois group, 它在一般的 p-adic representation 的像是有限的。怎樣的 representation 會有夠大的像呢？過去我研究專注的 field of norms functor 便給予一個對應到 Nottingham group 的 representation. 另一個方向便是考慮 tree representation. 其中的 Branch groups, 是最新發現可能的目標。

Branch groups 是一個近年來才被探討的理論。我們的研究方向首先是專注 Branch groups 的理論作更深入的了解。除了對 abstract branch groups 的理論我們有所了解外，我們也要參考一些已知 branch group 與 Galois representation 關係的文獻。過去有 Odoni [22] 和 Stoll [24] 處理了 x^2+1 的疊代情形。如何參考他們的方法處理 p-adic 以及 power series 的情形，應該是一個很有幫助的研究方向。

有關於 Lubin's Conjecture, Berger [2] 利用了 p-adic Hodge theory 處理了 full 的情形。過去我們曾利用 field of norm 處理了更一般的情形 [17]。由於 [17] 的處理方式是將 Galois group 考慮成 Nottingham group 的表現理論，我們也探討 [2] 是否和 Branch group 有關。這一方面在最後結果探討上, 我附上在 Communications in Algebra 發表有關 ramification groups 的相關內容.

5 結果與討論

最後附上在 Communications in Algebra 發表有關 ramification groups 的相關內容.

The proofs of our results are based on the following ([21, Theorem 4 (iv)]).

Lemma 1. *Let L/K be an abelian extension and let G denote the Galois group* $\mathrm{Gal}(L/K)$*. If K is of characteristic p, then the mapping $\sigma \to \sigma^p$ maps $G(n)$ into $G(pn)$, for all $n \in \mathbb{N}$.*

5.1 $\mathbb{Z}_p$-extensions

Given $\sigma \in \mathcal{N}(k)$, write $\lim_{n\to\infty}(i_n(\sigma)/p^n) = (p/(p-1))e$. It is well-known that when e is finite the closed cyclic group generated by σ corresponds to a $\mathbb{Z}_p$-extension of characteristic 0 (see for instance [9]). In this case, it's clear that $\lim_{n\to\infty}(i_n(\sigma)/i_{n-1}(\sigma)) = p$. That is $\mathrm{Height}(\sigma) = 1$. In this subsection, we first show that for the case $\sigma \in \mathcal{N}(k)$ with $\mathrm{Height}(\sigma) = 1$, the closed cyclic group generated by σ corresponds to a $\mathbb{Z}_p$-extension of characteristic 0. In fact, we have the following.

Theorem 2. *Suppose that $G \subseteq \mathcal{N}(k)$ is a closed subgroup which is isomorphic to $\mathbb{Z}_p$. Then the following are equivalent:*

1. *There exists an element $\sigma \in G$ with* $\mathrm{Height}(\sigma) = 1$
2. *The $\mathbb{Z}_p$-extension corresponding to G is of characteristic* 0*.*
3. *For every nonidentity $\sigma \in G$, $\frac{i_{n+1}(\sigma)-i_n(\sigma)}{i_n(\sigma)-i_{n-1}(\sigma)} = p$ for all n sufficiently large.*

Proof. We prove this by contradiction. Suppose that the corresponding $\mathbb{Z}_p$-extension is of characteristic p. Then it is also true that the $\mathbb{Z}_p$-extension corresponding to the closed subgroup H generated by σ^{p^n} is of characteristic p. By considering the ramification groups of H, we have $\sigma^{p^n} \in H[i_n(\sigma)] \setminus H[i_n(\sigma)+\varepsilon]$ and $\sigma^{p^{n+1}} \in H[i_{n+1}(\sigma)] \setminus H[i_{n+1}(\sigma)+\varepsilon]$, $\forall \varepsilon > 0$. Therefore $\phi_H(i_n(\sigma))$ and $\phi_H(i_{n+1}(\sigma))$ are upper ramification breaks of H and hence we can apply Lemma 1 to get $\sigma^{p^{n+1}} \in H(p\,\phi_H(i_n(\sigma)))$. In other words,

$$\phi_H(i_{n+1}(\sigma)) = i_n(\sigma) + \frac{i_{n+1}(\sigma) - i_n(\sigma)}{p} \geq p\,\phi_H(i_n(\sigma)) = p\,i_n(\sigma), \forall n \in \mathbb{N}.$$

This says that

$$i_{n+1}(\sigma) \geq (p^2 - p + 1) i_n(\sigma), \forall n \in \mathbb{N},$$

and hence contradicts the assumption that $\lim_{n\to\infty} \frac{i_n(\sigma)}{i_{n-1}(\sigma)} = p$.

Now let G be a subgroup of $\mathscr{N}(k)$ and $\sigma \in G$ be a nonidentity element. The closed subgroup H generated by σ is of finite index in G. Since H corresponds to a characteristic 0 $\mathbb{Z}_p$-extension, G also corresponds to a $\mathbb{Z}_p$-extension of characteristic 0. On the other hand, when G corresponds to a $\mathbb{Z}_p$-extension of characteristic 0, the main theorem in [17] says that for every nonidentity element $\sigma \in G$, the equation

$$\frac{i_{n+1}(\sigma) - i_n(\sigma)}{i_n(\sigma) - i_{n-1}(\sigma)} = p$$

holds for all n sufficiently large. This completes the proof. □

5.2 $\mathbb{Z}_p \times \mathbb{Z}_p$-extensions

In this subsection we extend the result of the previous subsection to the case that $G \subseteq \mathscr{N}(k)$ is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$. In this case we show that there exists a height 2 element in G if and only if the $\mathbb{Z}_p \times \mathbb{Z}_p$-extension corresponding to G is of characteristic 0. In fact, we have the following.

Theorem 3. *Suppose that $G \subseteq \mathscr{N}(k)$ is a closed subgroup which is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$. Then the following are equivalent:*

1. *There exists an element $\sigma \in G$ with* $\text{Height}(\sigma) = 2$.
2. *The $\mathbb{Z}_p \times \mathbb{Z}_p$-extension corresponding to G is of characteristic* 0.
3. *For every nonidentity $\sigma \in G$, $\frac{i_{n+1}(\sigma)-i_n(\sigma)}{i_n(\sigma)-i_{n-1}(\sigma)} = p^2$ for all n sufficiently large.*

Proof. Let G be a closed subgroup of $\mathscr{N}(k)$ which is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$ and suppose that there exists an element $\sigma \in G$ such that $\lim_{n\to\infty} i_n(\sigma)/i_{n-1}(\sigma) = p^2$. Again, we use method of contradiction to show that the $\mathbb{Z}_p \times \mathbb{Z}_p$-extension corresponding to G is of characteristic 0. First, suppose that the corresponding $\mathbb{Z}_p \times \mathbb{Z}_p$-extension is of characteristic p. Then for any two linearly independent elements $\sigma, \tau \in G$, since $\langle \sigma, \tau \rangle$ is a finite index subgroup of G (we use $\langle \sigma, \tau \rangle$ to denote the closed subgroup of G generated by σ and τ), the field extension corresponding to $\langle \sigma, \tau \rangle$ is also a characteristic p field extension. Similarly, for $m, n \in \mathbb{N}$, the $\mathbb{Z}_p \times \mathbb{Z}_p$ extension corresponding to $\langle \sigma^{p^n}, \tau^{p^m} \rangle$ is also of characteristic p.

For a given height 2 element $\sigma \in G$, we divide the growth of its depth into following cases.

1. For every $N \in \mathbb{N}$, there exist $n, m > N$ and $\tau \in G$ such that $i_n(\sigma) < i_m(\tau) < i_{m+1}(\tau) \leq i_{n+1}(\sigma)$.

2. For every $N \in \mathbb{N}$, there exist $n, m > N$ and $\tau \in G$ such that $i_n(\sigma) < i_m(\tau) < i_{n+1}(\sigma) < i_{n+2}(\sigma) \leq i_{m+1}(\tau)$.

3. There exists N such that there is neither $m, n > N$ nor any nonidentity $\tau \in G$ such that $i_n(\sigma) < i_m(\tau) < i_{n+1}(\sigma)$.

4. There exists $m, n \in \mathbb{N}$ and a nonidentity $\tau \in G$ such that $i_{n+j}(\sigma) < i_{m+j}(\tau) < i_{n+j+1}(\sigma)$, for all $j \in \mathbb{N}$.

For the case (1), notice that the field extension corresponding to the closed subgroup $H = \langle \sigma^{p^n}, \tau^{p^m} \rangle$ is also of characteristic p. By considering the lower ramification subgroups of H, we have

$$\begin{aligned} H[i_n(\sigma)] = \langle \sigma^{p^n}, \tau^{p^m} \rangle &\supsetneq H[i_n(\sigma)+1] = \cdots = H[i_m(\tau)] = \langle \sigma^{p^{n+1}}, \tau^{p^m} \rangle \\ &\supsetneq H[i_m(\tau)+1] = \cdots = H[i_{m+1}(\tau)] = \langle \sigma^{p^{n+1}}, \tau^{p^{m+1}} \rangle. \end{aligned}$$

Therefore,

$$\phi_H(i_m(\tau)) = i_n(\sigma) + \frac{i_m(\tau) - i_n(\sigma)}{p}$$

and

$$\phi_H(i_{m+1}(\tau)) = i_n(\sigma) + \frac{i_m(\tau) - i_n(\sigma)}{p} + \frac{i_{m+1}(\tau) - i_m(\tau)}{p^2}.$$

Since $\tau^{p^m} \in H[i_m(\tau)] = H(\phi_H(i_m(\tau)))$ and

$$\tau^{p^{m+1}} \in H(\phi_H(i_{m+1}(\tau))) \setminus H(\phi_H(i_{m+1}(\tau)) + \varepsilon),\ \forall \varepsilon > 0,$$

Lemma 1 says

$$i_n(\sigma) + \frac{i_m(\tau) - i_n(\sigma)}{p} + \frac{i_{m+1}(\tau) - i_m(\tau)}{p^2} \geq p(i_n(\sigma) + \frac{i_m(\tau) - i_n(\sigma)}{p}).$$

Therefore by $i_{n+1}(\sigma) \geq i_{m+1}(\tau)$ and $i_m(\tau) > i_n(\sigma)$, we have

$$i_{n+1}(\sigma) \geq (p^2 - p + 1) i_m(\tau) + (p^3 - 2p^2 + p) i_n(\sigma) > (p^3 - p^2 + 1) i_n(\sigma).$$

Since for every $N \in \mathbb{N}$, this is true for some $n > N$, it contradicts the assumption that $\lim_{n\to\infty} \frac{i_n(\sigma)}{i_{n-1}(\sigma)} = p^2$.

For case (2), by considering the lower ramification subgroups of H, we have

$$\begin{aligned} H[i_n(\sigma)] = \langle \sigma^{p^n}, \tau^{p^m} \rangle &\supsetneq H[i_n(\sigma)+1] = \cdots = H[i_m(\tau)] = \langle \sigma^{p^{n+1}}, \tau^{p^m} \rangle \\ &\supsetneq H[i_m(\tau)+1] = \cdots = H[i_{n+1}(\sigma)] = \langle \sigma^{p^{n+1}}, \tau^{p^{m+1}} \rangle \\ &\supsetneq H[i_{n+1}(\sigma)+1] = \cdots = H[i_{n+2}(\sigma)] = \langle \sigma^{p^{n+2}}, \tau^{p^{m+1}} \rangle. \end{aligned}$$

Therefore

$$\phi_H(i_{n+2}(\sigma)) = \phi_H(i_{n+1}(\sigma)) + \frac{i_{n+2}(\sigma) - i_{n+1}(\sigma)}{p^3}.$$

By Lemma 1, $\phi_H(i_{n+1}(\sigma)) \geq p\,\phi_H(i_n(\sigma)) = p\,i_n(\sigma)$. Since $\sigma^{p^{n+2}} \notin H[i_{n+2}(\sigma) + \varepsilon]$, $\forall \varepsilon > 0$, we have $\phi_H(i_{n+2}(\sigma)) \geq p\,\phi_H(i_{n+1}(\sigma))$. This implies

$$\frac{i_{n+2}(\sigma) - i_{n+1}(\sigma)}{p^3} \geq (p-1)\phi_H(i_{n+1}(\sigma)) \geq (p-1)p\,i_n(\sigma),$$

and hence

$$i_{n+2}(\sigma) \geq (p^5 - p^4) i_n(\sigma) + i_{n+1}(\sigma).$$

Since for every $N \in \mathbb{N}$, this is true for some $n > N$, it contradicts the assumption that $\lim_{n\to\infty} \frac{i_{n+1}(\sigma)}{i_n(\sigma)} = p^2$.

For the case (3), there exists $N \in \mathbb{N}$ such that

$$G[i_n(\sigma)] \supsetneq G[i_n(\sigma)+1] = \cdots = G[i_{n+1}(\sigma)], \forall n > N.$$

Now let $H = G[i_n(\sigma)]$. Then by the contrapositive assumption, the field extension corresponding to H is also of characteristic p. Since $\phi_H(i_{n+1}(\sigma)) = i_n(\sigma) + (1/p^2)(i_{n+1}(\sigma) - i_n(\sigma))$, again by Lemma 1 we have $i_n(\sigma) + (1/p^2)(i_{n+1}(\sigma) - i_n(\sigma)) \geq p i_n(\sigma)$ and hence

$$i_{n+1}(\sigma) \geq (p^3 - p^2 + 1) i_n(\sigma) \geq (p^2+1) i_n(\sigma).$$

This is true for all $n > N$, and hence it contradicts the assumption that $\text{Height}(\sigma) = 2$.

For the case (4), for every $j \in \mathbb{N}$, let $H = \langle \sigma^{p^{n+j}}, \tau^{p^{m+j}} \rangle$. Considering the ramification subgroups of H,

$$H[i_{n+j}(\sigma)] \supsetneq H[i_{m+j}(\tau)] \supsetneq H[i_{n+j+1}(\sigma)] \supsetneq H[i_{m+j+1}(\tau)] \supsetneq H[i_{n+j+2}(\sigma)],$$

we have

$$\phi_H(i_{n+j+1}(\sigma)) = i_{n+j}(\sigma) + \frac{i_{m+j}(\tau) - i_{n+j}(\sigma)}{p} + \frac{i_{n+j+1}(\sigma) - i_{m+j}(\tau)}{p^2}$$

and

$$\phi_H(i_{n+j+2}(\sigma)) = \phi_H(i_{n+j+1}(\sigma)) + \frac{i_{m+j+1}(\tau) - i_{n+j+1}(\sigma)}{p^3} + \frac{i_{n+j+2}(\sigma) - i_{m+j+1}(\tau)}{p^4}.$$

Again by the assumption that H corresponds to a field extension of characteristic p, we have $\phi_H(i_{n+j+2}(\sigma)) \geq p\,\phi_H(i_{n+j+1}(\sigma))$ and $\phi_H(i_{n+j+1}(\sigma)) \geq p\,\phi_H(i_{n+j}(\sigma))$. Using the fact that $\phi_H(i_{n+j}(\sigma)) = i_{n+j}(\sigma)$, we deduce that

$$\frac{i_{m+j+1}(\tau) - i_{n+j+1}(\sigma)}{p^3} + \frac{i_{n+j+2}(\sigma) - i_{m+j+1}(\tau)}{p^4} \geq (p-1)p\, i_{n+j}(\sigma),$$

and hence we have

$$i_{n+j+2}(\sigma) \geq (p^6 - p^5) i_{n+j}(\sigma) + p\, i_{n+j+1}(\sigma) - (p-1) i_{m+j+1}(\tau).$$

Finally, using the fact that $i_{m+j+1}(\tau) < i_{n+j+2}(\sigma)$, we deduce that

$$i_{n+j+2}(\sigma) \geq (p^5 - p^4) i_{n+j}(\sigma) + i_{n+j+1}(\sigma).$$

This is a contradiction to the assumption that

$$\lim_{n\to\infty} i_{n+2}(\sigma)/i_n(\sigma) = p^4, \lim_{n\to\infty} i_{n+1}(\sigma)/i_n(\sigma) = p^2,$$

and hence completes the proof of showing that if there is an element of height 2 in G, then the $\mathbb{Z}_p \times \mathbb{Z}_p$-extension corresponding to G is of characteristic 0.

Finally, the main theorem in [17] shows that the statement (2) and statement (3) are equivalent. This completes the proof. $\square$

References

[1] Benedetto, R., Faber, X., Hutz, B., Juul, J. & Yasufuku, Y.: A large arboreal Galois representation for a cubic postcritically finite polynomial, Res. Number Theory.

[2] L. Berger, Lubin's conjecture for full p-adic dynamical systems, Publications mathématiques de Besançon (2016), p. 19-24

[3] N. Boston, Tree representations of Galois groups, preprint.

[4] N. Boston and R. Jones, Arboreal Galois representations, *Geom. Dedicata* 124(1) (2007), 27-35.

[5] A. M. Brunner and S. Sidki, The generation of $\mathbf{GL}_n(\mathbb{Z})$ by finite state automata, *Internat. J. Algebra and Comput.* 8:1 (1998), 127-139.

[6] A. M. Brunner, S. Sidki and A. C. Viera, A just-nonsolvable torsion-free group defined on the binary tree, *J. Alg.* 211 (1999), 99-114.

[7] R. Camina, The Nottingham group, *New horizons in pro-p-groups*, Progress in mathematics **184** (Birkhauser, Boston, 2000).

[8] I. Fesenko, On just infinite pro-p-groups and arithmetically profinite extensions of local fields, *J. Reine Angew. Math.* 517 (1999), 61–80.

[9] I. Fesenko and M. du Sautoy, Where the wild things are: ramification groups and the Nottingham group, *New horizons in pro-p-groups*, Progress in mathematics **184** (Birkhauser, Boston, 2000).

[10] J.-M. Fontaine and B. Mazur, Geometric Galois representations, in "Elliptic curves and modular forms, Proceedings of a conference held in Hong Kong, December 18-21, 1993," International Press, Cambridge, MA and Hong Kong.

[11] R. I. Grigorchuk, Just Infinite Branch Groups, *New horizons in pro-p-groups*, Progress in mathematics **184** (Birkhauser, Boston, 2000).

[12] R.I. Grigorchuk, On the Burnside problem for periodic groups, *Func. Anal. Appl.* 14 (1980), 41-43.

[13] R. I. Grigorchuk, On Milnor's problem on group growth, *Soviet Math. Dokl.* 28 (1983),23-26.

[14] N. Gupta and l. Fabrykowski, On groups with sub-exponential growth function II, *J. Indian Math. Soc. (N. S.)* 56:1-4 (1991), 217-228.

[15] N. Gupta and S. Sidki, On the Burnside problem for periodic groups, *Math. Z.* 182 (1983), 385-388.

[16] Grigorchuk, R.I.: Just Infinite Branch Groups, Chapter 4 in New Horizons in pro-p Groups.

[17] L.-C. Hsia and H.-C. Li, Ramification filtrations of certain abelian Lie extensions of local fields, *J. Number Theory* 168 (2016) 135–153.

[18] H.-C. Li, On the Heights of elements in the Nottingham Group, COMMUNICATIONS IN ALGEBRA 2018, VOL. 46, NO. 11, 4777−4786

[19] J. Lubin, Nonarchimedean dynamical systems, *Compos. Math.* 94 (1994) 321–346.

[20] J. Lubin and J. Tate, Formal complex multiplication in local fields, *Ann. of Math.* (2) 81 (1965) 380–387.

[21] M. A. Marshall, Ramification groups of abelian local field extensions, *Canad. J. Math.* **XXIII** (2) (1971), pp. 271–281.

[22] Odoni, R.W.K.: Realising Wreath Products of Cyclic Groups as Galois Groups, MATHEMATIKA, 35 (1988), 101–113.

[23] Rotman, J: An Introduction to the Theory of Groups.

[24] M.Stoll, Galois groups over Q of some iterated polynomials, *Arch. Math. (Basel)* 59 (1992), no. 3, 239−244.

106年度專題研究計畫成果彙整表

計畫主持人：李華介	**計畫編號：**106-2115-M-003-001-
計畫名稱：非阿基米德動態系統與枝群	

成果項目					量化	單位	質化（說明：各成果項目請附佐證資料或細項說明，如期刊名稱、年份、卷期、起訖頁數、證號...等）
國內	學術性論文	期刊論文			0	篇	
		研討會論文			0		
		專書			0	本	
		專書論文			0	章	
		技術報告			0	篇	
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		
			新型/設計專利		0		
		商標權			0		
		營業秘密			0		
		積體電路電路布局權			0		
		著作權			0		
		品種權			0		
		其他			0		
	技術移轉	件數			0	件	
		收入			0	千元	
國外	學術性論文	期刊論文			1	篇	On the Heights of elements in the Nottingham Group, COMMUNICATIONS IN ALGEBRA, 2018, VOL. 46, NO. 11, 4777－4786
		研討會論文			0		
		專書			0	本	
		專書論文			0	章	
		技術報告			0	篇	
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		
			新型/設計專利		0		
		商標權			0		
		營業秘密			0		
		積體電路電路布局權			0		

		著作權	0		
		品種權	0		
		其他	0		
	技術移轉	件數	0	件	
		收入	0	千元	
參與計畫人力	本國籍	大專生	0	人次	
		碩士生	1		整理資料, 文獻.
		博士生	0		
		博士後研究員	0		
		專任助理	0		
	非本國籍	大專生	0		
		碩士生	0		
		博士生	0		
		博士後研究員	0		
		專任助理	0		
其他成果 （無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）					

科技部補助專題研究計畫成果自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現（簡要敘述成果是否具有政策應用參考價值及具影響公共利益之重大發現）或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估
■達成目標
□未達成目標（請說明，以100字為限）
　□實驗失敗
　□因故實驗中斷
　□其他原因
說明：

2. 研究成果在學術期刊發表或申請專利等情形（請於其他欄註明專利及技轉之證號、合約、申請及洽談等詳細資訊）
論文：■已發表　□未發表之文稿　□撰寫中　□無
專利：□已獲得　□申請中　■無
技轉：□已技轉　□洽談中　■無
其他：（以200字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性，以500字為限）
本結果對了解 Nottingham groups 有一定的幫助.

4. 主要發現
本研究具有政策應用參考價值：■否　□是，建議提供機關
（勾選「是」者，請列舉建議可提供施政參考之業務主管機關）
本研究具影響公共利益之重大發現：□否　□是
說明：（以150字為限）