科技部補助專題研究計畫成果報告
期末報告

關於動態系統模空間裏特殊子簇相關算術問題之研究(第2年)

計 畫 類 別 ： 個別型計畫
計 畫 編 號 ： MOST 106-2115-M-003-014-MY2
執 行 期 間 ： 107年08月01日至108年07月31日
執 行 單 位 ： 國立臺灣師範大學數學系（所）

計 畫 主 持 人： 夏良忠

計畫參與人員： 碩士班研究生-兼任助理：石毓萱
碩士班研究生-兼任助理：高智強
碩士班研究生-兼任助理：鍾明廷
博士班研究生-兼任助理：孫維良

報 告 附 件 ： 移地研究心得報告
出席國際學術會議心得報告

中 華 民 國 108 年 10 月 31 日

中 文 摘 要 ： (1) 我們考慮帶有參數的海儂映射族：$H_{t} : \AA^{2} \to \AA^{2}$ 其中
$H_{t}(x, y) = (y + f_{t}(x), x)$ 以及 $f_{t}(x) \in K[x, t]$ 為 x 次數至少為 2 的多項式。
令 $\Sigma(\bfP)$ 表初始點 $\bfP \in \AA^{2}(K[t])$ 在參數為 t 時在 H_{t} 為週期點的參數。我們希望了解找出 $\bfP$ 以及 H 所滿足的條件使得 $\Sigma(\bfP)$ 為一個無窮集合。

(2) 令 K 是定義於數域上平滑、不可約的代數曲線上的有理函數體。令 f 是形如
$f(x)=x^p+c$ 的多項式，其中 p 為質數而 $c\in K$，且令 $\beta\in \Kbar$。對於所有的正整數 n，Galois 群 $G_n(\beta)=\Gal(K(f^{-n}(\beta))/K(\beta))$ 可嵌入 $[C_p]^n$ -- 循環群 C_p 的第 n 層輪積 (wreath product)。我們證明若 f 不等同平凡的 (not isotrivial)，除非 β 是後臨界點或週期點，否則 $[[C_p]^\infty:G_\infty(\beta)]<\infty$ 。我們同時也證給定兩個不等同平凡的多項式 $f_1(x)=x^p+c_1$ 與 $f_2(x)=x^p+c_2$，所得的擴張體 $\bigcup_{n=1}^\infty K(f_1^{-n}(\beta))$ 與 $\bigcup_{n=1}^\infty K(f_2^{-n}(\beta))$ 在 K 的有限擴張之後互斥。

中 文 關 鍵 詞 ： 算術動態系統、不可能相交問題、一維參數族、海儂映射、週期點、動態相關的、Galois 樹叢表現、迭代 Galois 群

英 文 摘 要 ： (1) We consider a 1-parameter family $H_{t} : \AA^{2} \to \AA^{2}$ of Hénon maps, where $H_{t}(x, y) = (y + f_{t}(x), x)$ and $f_{t}(x) \in K[x, t]$ with the degree (in x) of $f_{t}(x)$ being at least 2. Let $\Sigma(\bfP)$ denotes the set of parameters $t\in \Kbar$ where the specialized point $P_{t} \in \AA^{2}(K[t])$ is periodic for the specialized H\'enon map $H_{t}.$ We're interested in conditions satisfied by $\bfP$ and H under which the set of periodic parameters $\Sigma(\bfP)$ is an infinite set.

(2) Let K be the function field of a smooth, irreducible curve defined over $\Qbar$. Let $f\in K[x]$ be of the form $f(x)=x^p+c$ for p prime, and let $\beta\in \Kbar$. For all $n\in\mathbb{N}\cup\{\infty\}$, the Galois groups $G_n(\beta)=\Gal(K(f^{-n}(\beta))/K(\beta))$ embed into $[C_p]^n$, the n-fold wreath product of the cyclic group C_p. We show that if f is not isotrivial, then $[[C_p]^\infty:G_\infty(\beta)]<\infty$ unless β is postcritical or periodic. We are also able to prove that if $f_1(x)=x^p+c_1$ and $f_2(x)=x^p+c_2$ are two such polynomials, then the fields $\bigcup_{n=1}^\infty K(f_1^{-n}(\beta))$ and $\bigcup_{n=1}^\infty K(f_2^{-

n}(\beta))$ are disjoint over a finite extension of K.

英文關鍵詞：arithmetic dynamics, unlikely intersection questions, 1-parameter family, Hénon maps, periodic points, dynamically related, arboreal Galois representations, iterated Galois groups

一、中英文摘要與關鍵詞:

一、中文摘要:

(1) 我們考慮帶有參數的海儂映射族：$H_t : \mathbb{A}^2 \to \mathbb{A}^2$ 其中 $H_t(x,y) = (y + f_t(x), x)$ 以及 $f_t(x) \in K[x,t]$ 為 x 次數至少為 2 的多項式。令 $\Sigma(\mathbf{P})$ 表初始點 $\mathbf{P} \in \mathbb{A}^2(K[t])$ 在參數為 t 時在 H_t 為週期點的參數。我們希望了解找出 $\mathbf{P}$ 以及 H 所滿足的條件使得 $\Sigma(\mathbf{P})$ 為一個無窮集合。

(2) 令 K 是定義於數域上平滑、不可約的代數曲線上的有理函數體。令 f 是形如 $f(x) = x^p + c$ 的多項式，其中 p 為質數而 $c \in K$，且令 $\beta \in \bar{K}$。對於所有的正整數 n，Galois 群 $G_n(\beta) = \mathrm{Gal}(K(f^{-n}(\beta))/K(\beta))$ 可嵌入 $[C_p]^n$ – 循環群 C_p 的第 n 層輪積 (wreath product) 。我們證明若 f 不等同平凡的 (not isotrivial)，除非 β 是後臨界點或週期點，否則 $[[C_p]^\infty : G_\infty(\beta)] < \infty$ 。我們同時也證給定兩個不等同平凡的多項式 $f_1(x) = x^p + c_1$ 與 $f_2(x) = x^p + c_2$，所得的擴張體 $\bigcup_{n=1}^{\infty} K(f_1^{-n}(\beta))$ 與 $\bigcup_{n=1}^{\infty} K(f_2^{-n}(\beta))$ 在 K 的有限擴張之後互斥。

關鍵詞: 算術動態系統、不可能相交問題、一維參數族、海儂映射、週期點、動態相關的、Galois 樹叢表現、迭代 Galois 群

Abstract.

(1) We consider a 1-parameter family $H_t : \mathbb{A}^2 \to \mathbb{A}^2$ of Hénon maps, where $H_t(x, y) = (y + f_t(x), x)$ and $f_t(x) \in K[x, t]$ with the degree (in x) of $f_t(x)$ being at least 2. Let $\Sigma(\mathbf{P})$ denotes the set of parameters $t \in \bar{K}$ where the specialized point $P_t \in \mathbb{A}^2(K[t])$ is periodic for the specialized Hénon map H_t. We're interested in conditions satisfied by $\mathbf{P}$ and H under which the set of periodic parameters $\Sigma(\mathbf{P})$ is an infinite set.
(2) Let K be the function field of a smooth, irreducible curve defined over $\bar{\mathbb{Q}}$. Let $f \in K[x]$ be of the form $f(x) = x^p + c$ for p prime, and let $\beta \in \bar{K}$. For all $n \in \mathbb{N} \cup \{\infty\}$, the Galois groups $G_n(\beta) = \mathrm{Gal}(K(f^{-n}(\beta))/K(\beta))$ embed into $[C_p]^n$, the n-fold wreath product of the cyclic group C_p. We show that if f is not isotrivial, then $[[C_p]^\infty : G_\infty(\beta)] < \infty$ unless β is postcritical or periodic. We are also able to prove that if $f_1(x) = x^p + c_1$ and $f_2(x) = x^p + c_2$ are two such polynomials, then the fields $\bigcup_{n=1}^{\infty} K(f_1^{-n}(\beta))$ and $\bigcup_{n=1}^{\infty} K(f_2^{-n}(\beta))$ are disjoint over a finite extension of K.

Key Words : arithmetic dynamics, unlikely intersection questions, 1-parameter family, Hénon maps, periodic points, dynamically related, arboreal Galois representations, iterated Galois groups

二、前言：

In this two year project, we continue to study the problems on the *special points/varieties* arising from algebraic families of dynamical systems. The main results achieved include (i) the study of periodic points in a one- parameter family of Hénon maps; (ii) finite index theorems for iterated Galois groups of unicritical polynomials.

The background for results in (i) have been discussed in the first year report of this project. For a given one-parameter family of Hénon maps $\mathbf{H} = \{H_t\} : \mathbb{A}^2 \to \mathbb{A}^2$ of the affine plane and a family of initial point $\mathbf{P} := \{P_t = (a(t), b(t))\}$ parameterized by points $t \in Y(\bar{K})$ of algebraic curve over a global field K. The main question is to study whether or not the set of "periodic parameters"

$$\Sigma(\mathbf{P}) = \{t \in Y(\bar{K}) \mid P_t \text{ is periodic for } H_t\}$$

is an infinite set.

For (ii), we're concerned with the following question: Let K be a field. Let $f \in K(x)$ with $d = \deg f \geq 2$ and let $\beta \in \mathbb{P}^1(\bar{K})$. For $n \in \mathbb{N}$, let $K_n(f, \beta) = K(f^{-n}(\beta))$ be the field obtained by adjoining the nth preimages of β under f to $K(\beta)$ (we declare that $K(\infty) = K$). Set $K_\infty(f, \beta) = \bigcup_{n=1}^{\infty} K_n(f, \beta)$. For $n \in \mathbb{N} \cup \{\infty\}$, define $G_n(f, \beta) = \mathrm{Gal}(K_n(f, \beta)/K(\beta))$. We will write $G_n(\beta)$ and $K_n(\beta)$, suppressing the dependence on f if there is no ambiguity. Put $G_\infty(\beta) = \mathrm{Gal}(K_\infty(f, \beta)/K)$ which is the inverse limit of $G_n(f, \beta)$. The question we're interested in is how large is the group $G_\infty(\beta)$? There are obvious special cases such that $G_\infty(\beta)$ is smaller than expected. Are these special cases the only instances such that $G_\infty(\beta)$ is smaller than expected?

三、研究目的：

A motivation for (i) comes from our previous work on unlikely intersections for family of polynomial maps f_t on $\mathbb{P}^1$ and a marked point $c \in \mathbb{P}^1(\mathbb{C})$. Unlike the case for one-parameter family of polynomial maps where the set of periodic parameters $\Sigma(c)$ is always infinite, it may happen that $\Sigma(\mathbf{P})$ can be finite in the case of one-parameter family of Hénon maps. A heuristic reasoning can be seen as follows. The coordinates of the n-th orbit $H^n(\mathbf{P})$ of $\mathbf{P}$ under the action of H can be written as (A_n, A_{n-1}) where A_n is a non-linear recurrence sequence of rational functions on Y. Then the parameter $t \in \Sigma(\mathbf{P})$ is a point $t \in Y$ which is a solution to the equations : $A_n(t) = a(t), A_{n-1}(t) = b(t)$. In general, one can not expect that there are many solutions to such equations as n ranges over the set of positive integers unless the family of Hénon maps $\mathbf{H}$ and the given point $\mathbf{P}$ are special. This is similar to the GCD problem in linear recurrence sequences which is an active research area in recent years. As our sequence $\{A_n\}$ is a non-linear recurrence sequence, most of the techniques used in the linear case can not be applied. There are new difficulties here.

For families of Hénon maps, in a joint work with S. Kawaguchi we raise the question of determining the conditions satisfied by the maps $\mathbf{H}$ as well as $\mathbf{P}$ as follows.

Question *Let K be an algebraically closed field of characteristic zero. Let $\mathbf{H}\colon \mathbb{A}^2 \to \mathbb{A}^2$ be a Hénon map over $\mathbb{A}^1 = \mathrm{Spec}(K[t])$, which we regard as a family $(H_t)_{t\in K}$ parametrized by t. Put $\mathrm{Per}_{\mathbb{A}^2\times\mathbb{A}^1}(\mathbf{H}) := \{(P,t) \in \mathbb{A}^2(K) \times \mathbb{A}^1(K) \mid P \text{ is periodic with respect to } H_t\}$. Let $\mathcal{C}$ be an integral curve in $\mathbb{A}^2 \times \mathbb{A}^1$, and we assume that $\mathcal{C}(K) \cap \mathrm{Per}_{\mathbb{A}^2\times\mathbb{A}^1}(\mathbf{H})$ is infinite. Is it true that one of the following conditions must hold?*

(i) $\mathcal{C} \subseteq \mathbb{A}^2_t = \mathbb{A}^2 \times \{t\}$ for some $t \in \mathbb{A}^1(K)$, and there exists an involution σ_t of $\mathbb{A}^2$ over K such that $\sigma_t \circ H_t^m \circ \sigma_t = H_t^{-m}$ for some $m \geq 1$. Furthermore, $\mathcal{C}$ is contained in the set of fixed points of σ_t;

(ii) There exists a positive integer m such that $\mathcal{C}$ is an irreducible component of $\{(P,t) \in \mathbb{A}^2(K) \times \mathbb{A}^1(K) \mid H_t^m(P) = P\}$;

(iii) There exists an involution $\boldsymbol{\sigma}$ of $\mathbb{A}^2$ over $K[t]$ such that $\boldsymbol{\sigma}\circ\mathbf{H}^m\circ\boldsymbol{\sigma} = \mathbf{H}^{-m}$ for some $m \geq 1$ and $\mathcal{C}$ is contained in the set of fixed points of $\boldsymbol{\sigma}$.

We give a partial answer this question in [4].

The question studied in (ii) is related to the so-called arboreal representation initiated by Odoni [7]. The Galois group $G_\infty(\beta)$ defined in previous part of the report embeds into $\mathrm{Aut}(T_\infty^d)$, the automorphism group of an infinite d-ary rooted tree T_∞^d. Recently there has been much work on the problem of determining when the index $[\mathrm{Aut}(T_\infty^d) : G_\infty(\beta)]$ is finite. The group $G_\infty(\beta)$ is the image of an arboreal Galois representation, so this finite index problem is a natural analog in arithmetic dynamics of the finite index problem for the ℓ-adic Galois representations associated to elliptic curves, resolved by Serre's celebrated Open Image Theorem [8]. By work of Odoni [7], one expects that a generically chosen rational function has a surjective arboreal representation, i.e., that $[\mathrm{Aut}(T_\infty^d) : G_\infty(\beta)] = 1$.

We study the family of polynomials $f(x) = x^d + c$ for $c \in K$, which up to change of variables represents all polynomials with precisely one (finite) critical point. If the field K contains a primitive dth root of unity, then it is easy to show that for f in this family, $G_\infty(\beta)$ sits in $[C_d]^\infty$, the infinite iterated wreath product of the cyclic group C_d (with d elements). As $\mathrm{Aut}(T_n^d) \cong [S_d]^n$, this means that if $d \geq 3$, then $[\mathrm{Aut}(T_\infty^d) : [C_d]^\infty] = \infty$. Thus it is impossible for $G_\infty(\beta)$ to have finite index within this family (except when $d = 2$). However, this simply means that, given the constraint on the size of $G_\infty(\beta)$, we should ask a different finite index question. We turn to the problem of when $G_\infty(\beta)$ has finite index in $[C_d]^\infty$. We give an answer to the question in the paper [5]

四、研究方法、結果與討論：

(1) Concerning the question about the set of periodic parameters associated to families of Hénon maps and initial points, among other things obtained in [4], we prove the following theorem.

Theorem 1 *Let K be a field (of any characteristic). Let $\mathbf{H}$ be the family of Hénon maps given by $H_t(x,y) = (\delta y + f_t(x), x)$ where $\delta = 1$ or -1 and $f_t(-x) = f_t(x) \in K[t,x]$ is a polynomial of degree $d \geq 2$ in x. When $\delta = 1$, we*

assume that $f_t(x)$ is an even polynomial in x. Let $\mathbf{P} = (a(t), b(t)) \in \mathbb{A}^2(K[x])$. If $\mathbf{P}$ satisfies $\delta a(t) + b(t) = 0$ in $K[t]$, then $\Sigma(\mathbf{P})$ is an infinite set.

We note that for $\delta \in \{1, -1\}$, the map $\iota_\delta \colon \mathbb{A}^2 \to \mathbb{A}^2$ given by $\iota_\delta \colon (x, y) \mapsto (-\delta y, -\delta x)$ is actually an involution of the affine plane. Furthermore, the assumption on the polynomial $f_t(x)$ is equivalent to the fact that $\iota \circ \mathbf{H} \circ \iota = \mathbf{H}^{-1}$. The condition on $\mathbf{P}$ simply says that $\mathbf{P}$ is a family of points fixed by ι_δ. By introducing the notion of reduction modulo ideals in the polynomial ring $K[t]$, we are able to reduce the question of solutions to the two equations $A_n(t) = a(t), A_{n-1}(t) = b(t)$ into the zeros of the sequence of polynomials $\delta A_n(t) + A_{n-1}(t)$. We then estimate the bound of multiplicities of the periodic points to ensure there are infinite roots in this sequence of polynomials and thus proving the theorem.

(2) Before stating the main result in [5], we set some notation. The field K refers to a function field of transcendence degree 1 over its field of constants $\bar{\mathbb{Q}}$. In other words, K is the function field of a smooth, projective, irreducible curve C over $\bar{\mathbb{Q}}$. We say that $f \in K[x]$ is isotrivial if f is defined over $\bar{\mathbb{Q}}$ up to a change of variables, that is, if $\varphi^{-1} \circ f \circ \varphi \in \bar{\mathbb{Q}}[x]$ for some $\varphi \in \bar{K}[x]$ of degree 1. In the special case of a unicritical polynomial $f(x) = x^d + c \in K[x]$, we have that f is isotrivial if and only if $c \in \bar{\mathbb{Q}}$. We say $\beta \in \bar{K}$ is periodic for f if $f^n(\beta) = \beta$ for some $n \geq 1$, and we say β is preperiodic for f if $f^m(\beta)$ is periodic for some $m \geq 0$. Finally, we say that β is postcritical for f if $f^n(\alpha) = \beta$ for some $n \geq 1$ and some critical point α of f.

With this notation, our first main theorem is as follows.

Theorem 2 *Let p be prime, let $c \in K \smallsetminus \bar{\mathbb{Q}}$, let $f(x) = x^p + c \in K[x]$ and let $\beta \in K$. Then the following are equivalent:*

1. *The point β is neither periodic nor postcritical for f.*
2. *The group $G_\infty(\beta)$ has finite index in $[C_p]^\infty$.*

In the case where $p = 2$, this means that $G_\infty(\beta)$ has finite index in $\mathrm{Aut}(T_\infty^2)$. For larger p this index is infinite, as mentioned previously. It is fairly easy to see that the conditions on β in Theorem 2 are necessary. If β is periodic or postcritical, then $[[C_p]^\infty : G_\infty(\beta)] = \infty$ by a straightforward argument. Most of the paper is devoted to the showing that these conditions are sufficient.

Remark 3 *In general one needs to rule out postcritically finite (PCF) maps in order to obtain a finite index result. The reason we do not need to do this in Theorem 2 is that a PCF polynomial of the form $f(x) = x^p + c$ is automatically isotrivial. This is because c satisfies an equation of the form $f^n(c) = f^m(c)$ for some $n > m \geq 0$, and so $c \in \bar{\mathbb{Q}}$. For isotrivial polynomials the PCF distinction regains its importance; see Section* **??**.

One of the key steps in the proof of Theorem 2 is an eventual stability result. As is usual in arithmetic dynamics, we say that the pair (f, β) is eventually stable over the field K if the number of irreducible K-factors of $f^n(x) - \beta$ is uniformly bounded for all n.

Theorem 4 *Let p be prime. Let $f \in K[x]$ be a polynomial of the form $x^p + t$ where $t \notin \bar{\mathbb{Q}}$. Then for any non-periodic $\beta \in K$, the pair (f, β) is eventually stable over K.*

We also prove the following disjointness theorem for fields generated by inverse images of different points under different maps.

Theorem 5 *For $i = 1, \ldots, n$ let $f_i(x) = x^p + c_i \in K[x]$, where $c_i \notin \bar{\mathbb{Q}}$, and let $\alpha_i \in K$. Suppose that there are no distinct i, j with the property that (α_i, α_j) lies on a curve in $\bar{A}^2$ that is periodic under the action of $(x, y) \mapsto (f_i(x), f_j(y))$. For each i, let M_i denote $K_\infty(f_i, \alpha_i)$. Then*

$$\left[M_i \cap \left(\prod_{j \neq i} M_j \right) : K \right] < \infty.$$

Theorem 5 also has a natural interpretation as a finite index result across pre-image trees of several points.

Remark 6 *In light of Odoni's work, unicritical polynomials with degree $d \geq 3$ cannot be considered generic from the point of view of arboreal Galois theory (indeed, they are not a generic family in the moduli space of degree d polynomials in any reasonable sense). There are other families of polynomials and rational functions (such as postcritically finite maps) that arise as obstructions to any potential classification of finite index arboreal representation. One might hope that in these "exceptional" families, something similar to Theorem 2 could hold, in that a broad finite index result could be established for a natural overgroup other than $\mathrm{Aut}(T_\infty^d)$. This will be explored in future work.*

References

[1] D. Ghioca, L. C. Hsia and T. J. Tucker, *Preperiodic points for families of polynomials*, *Algebra and Number Theory*, **7, no. 3** (2013), 701-732.

[2] D. Ghioca, L.-C. Hsia, and T. J. Tucker, *Preperiodic points for families of rational maps*, Proc. Lond. Math. Soc. (3) **110** (2015), no. 2, 395–427.

[3] D. Ghioca, L. C. Hsia and T. J. Tucker, *Unlikely Intersection For Two-Parameter Families of Polynomials,*, preprint.

[4] L. C. Hsia and S. Kawaguchi, *Heights and periodic points for one-parameter families of Hénon maps* , submitted (2018),

[5] A. Bridy, J. Doyle, D. Ghioca, L. C. Hsia and T. J. Tucker, *Finite index theorems for iterated galois group of unicritical polynomials*, submitted (2019),

[6] L. C. Hsia and T. J. Tucker, *Greatest common divisors of iterates of polynomials*, Algebra and Number Theory **11, no. 6** (2017), 1437-1459.

[7] Odoni, R. W. K., *The Galois theory of iterates and composites of polynomials*, Proc. London Math. Soc. vol. 51 (3), (1985), 385-414.

[8] Serre, J. P., *Propriétés galoisiennes des points d'ordre fini des courbes elliptiques*, Invent. Math. **15** no. 4, (1972), 259–331.

[9] J. H. Silverman, *Common divisors of elliptic divisibilty sequences over function field,* Manuscripta Math. **114** (2004), 431-446.

[10] J. H. Silverman, *Generalized Greatest Common Divisors, Divisible Sequences, and Vojta's Conjecture for Blowups,* Monatsh. Math., **145** (2005), 333-350

[11] U. Zannier, *Some problems of unlikely intersections in arithmetic and geometry*, Annals of Mathematics Studies, vol. 181, Princeton University Press, Princeton, NJ, 2012. With appendixes by David Masser.

科技部專題研究計畫出國開會、訪問報告

計畫編號　：MOST 106-2115-M-003-014-MY2

執行期限　：106 年 8 月 1 日至 108 年 7 月 31 日

主持人　：夏良忠

執行機構及單位　：國立台灣師範大學數學系

一、緣起:

The PI was invited to give a talk at the conference on number theory and its applications which was held in Xian City from the 18th, August, 2018 to August 22, 2018. After the conference, the PI was also invited to visit the Department of Mathematics at Northwest University for three days.

二、訪問活動概述：

Because the main number theory group in Xian area is on analytic number theory under the lead by Professor Wengpen Zhang at Northwest University, the main theme of the conference is around analytic number theory and its applications. On the other hand, there are problems in arithmetic dynamics that are analytic number theoretical in nature. By participating this conference, I had a chance to communicate with several analytic number theorists in China, including Wengpen Zhang, Zefeng Xu and Jianya Liu.

The talk I gave is part of my joint work with Shu Kawaguchi on the arithmetic of families of Hénon maps. Specifically, I talk about conditions under which a given family of Hénon maps and initial points having an infinite set of periodic parameters.

The following is the title and abstract of my talk:

Title: Periodic points in a family of Hénon maps

Abstract: Let $\mathbf{H} : \mathbb{A}^2 \to \mathbb{A}^2$ be a Hénon map defined over the polynomial ring $\mathbb{C}[t]$ given by $\mathbf{H}(x, y) = (\delta y + f_t(x), x)$ where $f_t(x) \in \mathbb{C}[t, x]$ viewed as a polynomial (in x) defined over $\mathbb{C}[t]$ of degree $\deg_x f_t(x) \geq 2$. The map $\mathbf{H}$ can be viewed as a family of Hénon maps $\{H_\lambda(x, y)\}$ with $H_\lambda = (\delta y + f_\lambda(x), x)$ parameterized by $t = \lambda \in \mathbb{C}$. Given point $\mathbf{P} = (a(t), b(t)) \in \mathbb{A}^2(\mathbb{C}[t])$ which is also a family of points $P_\lambda = (a(\lambda), b(\lambda))$ with $\lambda \in \mathbb{C}$. Assume that $\mathbf{P}$ does not satisfy $\mathbf{H}^n(\mathbf{P}) = \mathbf{P}$ for any positive integer n where $\mathbf{H}^n = \mathbf{H} \circ \mathbf{H} \circ \cdots \circ \mathbf{H}$ (n-th iterates of $\mathbf{H}$). We're interested in whether or not the set

$$\Sigma(\mathbf{P}) := \{\lambda \in \mathbb{C} \mid H_\lambda^n(P_\lambda) = P_\lambda \quad \text{for some positive integer } n\}$$

is an infinite set. In general, it is not expected that $\Sigma(\mathbf{P})$ is infinite unless $\mathbf{H}$ is an *reversible* family and $\mathbf{P}$ satisfy some conditions. We'll discuss what these are in the talk. This is part of a joint work with Shu Kawaguchi.

Visiting Northwest University: One of the participants of the conference is Prof. Daqing Wan at UC, Irvine. Prof. Wan is a well-known expert on p-adic analysis, p-adic zeta functions. During my visit at Northwest University, Prof. Wan gave three interesting lectures on the newest development of p-adic zeta functions and exponential sums. We had many discussions on the p-adic zeta functions and related subject in dynamical systems over non-archimedean fields. Besides the topics on exponential sums and p-adic zeta functions, I also had a lot of conversations with Prof. Zhang on the possibility of applying techniques and results to the study of arithmetic dynamics. We look forward to chances to have collaborations in this direction in the future.

三、感謝：

I would like to thank MOST for the support and to my host for hospitality I received.

科技部專題研究計畫出國開會、訪問報告

計畫編號　：MOST 106-2115-M-003-014-MY2

執行期限　：106 年 8 月 1 日至 108 年 7 月 31 日

主持人　：夏良忠

執行機構及單位　：國立台灣師範大學數學系

一、緣起:

The PI was invited to give a talk at the conference on number theory and its applications which was held in Xian City from the 18th, August, 2018 to August 22, 2018. After the conference, the PI was also invited to visit the Department of Mathematics at Northwest University for three days.

二、訪問活動概述：

Because the main number theory group in Xian area is on analytic number theory under the lead by Professor Wengpen Zhang at Northwest University, the main theme of the conference is around analytic number theory and its applications. On the other hand, there are problems in arithmetic dynamics that are analytic number theoretical in nature. By participating this conference, I had a chance to communicate with several analytic number theorists in China, including Wengpen Zhang, Zefeng Xu and Jianya Liu.

The talk I gave is part of my joint work with Shu Kawaguchi on the arithmetic of families of Hénon maps. Specifically, I talk about conditions under which a given family of Hénon maps and initial points having an infinite set of periodic parameters.

The following is the title and abstract of my talk:

Title: Periodic points in a family of Hénon maps

Abstract: Let $\mathbf{H} : \mathbb{A}^2 \to \mathbb{A}^2$ be a Hénon map defined over the polynomial ring $\mathbb{C}[t]$ given by $\mathbf{H}(x, y) = (\delta y + f_t(x), x)$ where $f_t(x) \in \mathbb{C}[t, x]$ viewed as a polynomial (in x) defined over $\mathbb{C}[t]$ of degree $\deg_x f_t(x) \geq 2$. The map $\mathbf{H}$ can be viewed as a family of Hénon maps $\{H_\lambda(x, y)\}$ with $H_\lambda = (\delta y + f_\lambda(x), x)$ parameterized by $t = \lambda \in \mathbb{C}$. Given point $\mathbf{P} = (a(t), b(t)) \in \mathbb{A}^2(\mathbb{C}[t])$ which is also a family of points $P_\lambda = (a(\lambda), b(\lambda))$ with $\lambda \in \mathbb{C}$. Assume that $\mathbf{P}$ does not satisfy $\mathbf{H}^n(\mathbf{P}) = \mathbf{P}$ for any positive integer n where $\mathbf{H}^n = \mathbf{H} \circ \mathbf{H} \circ \cdots \circ \mathbf{H}$ (n-th iterates of $\mathbf{H}$). We're interested in whether or not the set

$$\Sigma(\mathbf{P}) := \{\lambda \in \mathbb{C} \mid H_\lambda^n(P_\lambda) = P_\lambda \quad \text{for some positive integer } n\}$$

is an infinite set. In general, it is not expected that $\Sigma(\mathbf{P})$ is infinite unless $\mathbf{H}$ is an *reversible* family and $\mathbf{P}$ satisfy some conditions. We'll discuss what these are in the talk. This is part of a joint work with Shu Kawaguchi.

Visiting Northwest University: One of the participants of the conference is Prof. Daqing Wan at UC, Irvine. Prof. Wan is a well-known expert on p-adic analysis, p-adic zeta functions. During my visit at Northwest University, Prof. Wan gave three interesting lectures on the newest development of p-adic zeta functions and exponential sums. We had many discussions on the p-adic zeta functions and related subject in dynamical systems over non-archimedean fields. Besides the topics on exponential sums and p-adic zeta functions, I also had a lot of conversations with Prof. Zhang on the possibility of applying techniques and results to the study of arithmetic dynamics. We look forward to chances to have collaborations in this direction in the future.

三、感謝：

I would like to thank MOST for the support and to my host for hospitality I received.

106年度專題研究計畫成果彙整表

計畫主持人：夏良忠	**計畫編號：**106-2115-M-003-014-MY2
計畫名稱：關於動態系統模空間裏特殊子簇相關算術問題之研究	

成果項目					量化	單位	質化 （說明：各成果項目請附佐證資料或細項說明，如期刊名稱、年份、卷期、起訖頁數、證號...等）
國內	學術性論文	期刊論文			0	篇	
		研討會論文			0		
		專書			0	本	
		專書論文			0	章	
		技術報告			2	篇	科技部期中、期末報告。
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		
			新型/設計專利		0		
		商標權			0		
		營業秘密			0		
		積體電路電路布局權			0		
		著作權			0		
		品種權			0		
		其他			0		
	技術移轉	件數			0	件	
		收入			0	千元	
國外	學術性論文	期刊論文			2	篇	Preprints (1) L.~C.~Hsia and S.~Kawaguchi, Heights and periodic points for one-parameter families of H\'enon maps, submitted. (2) A.~Bridy, J.~Doyle, D.~Ghioca, L.~C.~Hsia and T.~J.~Tucker, Finite index theorems for iterated galois group of unicritical polynomials, submitted.
		研討會論文			0		
		專書			0	本	
		專書論文			0	章	
		技術報告			0	篇	
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		
			新型/設計專利		0		

		商標權	0		
		營業秘密	0		
		積體電路電路布局權	0		
		著作權	0		
		品種權	0		
		其他	0		
	技術移轉	件數	0	件	
		收入	0	千元	
參與計畫人力	本國籍	大專生	0	人次	
		碩士生	3		師大碩士生：石毓萱、高智強、鍾明廷參與學習討論
		博士生	1		師大博士生：孫維良參與討論
		博士級研究人員	0		
		專任人員	0		
	非本國籍	大專生	0		
		碩士生	0		
		博士生	0		
		博士級研究人員	0		
		專任人員	0		
其他成果 （無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）					

科技部補助專題研究計畫成果自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現（簡要敘述成果是否具有政策應用參考價值及具影響公共利益之重大發現）或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估
   ■達成目標
   □未達成目標（請說明，以100字為限）
   　□實驗失敗
   　□因故實驗中斷
   　□其他原因
   說明：

2. 研究成果在學術期刊發表或申請專利等情形（請於其他欄註明專利及技轉之證號、合約、申請及洽談等詳細資訊）
   論文：□已發表　■未發表之文稿　□撰寫中　□無
   專利：□已獲得　□申請中　■無
   技轉：□已技轉　□洽談中　■無
   其他：（以200字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性，以500字為限）
   We study the question about the conditions under which a given family of Henon maps and initial points having an infinite set of periodic parameters. We showed that if the Henon maps and points enjoy certain symmetries, then there indeed exist infinite periodic parameters. We suspect that the non-existence of such symmetry is the only obstruction for the given family of Henon maps and points having infinite periodic parameters. We also study problems on iterated Galois groups.

4. 主要發現
   本研究具有政策應用參考價值：■否　□是，建議提供機關
   （勾選「是」者，請列舉建議可提供施政參考之業務主管機關）
   本研究具影響公共利益之重大發現：□否　□是
   說明：（以150字為限）