科技部補助專題研究計畫報告

光滑度量測度空間上的拉普拉斯算子

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本研究具有政策應用參考價值：■否 □是，建議提供機關
（勾選「是」者，請列舉建議可提供施政參考之業務主管機關）
本研究具影響公共利益之重大發現：□否 □是

中 華 民 國 109 年 10 月 30 日

中 文 摘 要 ： 我們參考且學習多位學者的工作，包括Grigor'yan, Kondratiev, Sun, Mastrolia, Monticelli, Punzo and Zhang等等。即在加權的體積增長條件或V的加權積分條件下，則不存在微分不等式的正弱解，或者非負解必為零。在這個工作，我們想推展他們的結果，並且在柯西黎曼流形上找相似的性質。我們先注重在加權的CR流形上，學習並研究具有初始條件的拋物微分不等式的非負弱解。

中 文 關 鍵 詞 ： 加權CR流形, 拋物微分不等式, 加權拉普拉斯算子

英 文 摘 要 ： We refer and study the works by Grigor'yan, Kondratiev, Sun, Mastrolia, Monticelli, Punzo and Zhang et al, which consider the weighted volume growth condition or the weighted integral of V, then there is nonexistence of positive weak solution of the differential inequality, or any nonnegative weak solution of the differential inequality must be zero. In this work, we try to extend these results and find the analogous properties in CR manifold. That is, we focus on closed weighted CR manifolds and study the nonnegative solution of the parabolic differential inequality with initial condition $u_0>0$.

英 文 關 鍵 詞 ： weighted CR manifold, parabolic differential inequality, weighted Laplacian

報告內容:

(一). 前言、研究目的:

In this work, we study the nonnegative solutions of the parabolic differential inequality

$$\begin{cases} u_t - \Delta_{b,p} u \geq V u^q & \text{in } M \times (0,\infty), \\ u = u_0 & \text{in } M \times \{0\}, \end{cases} \tag{1}$$

on a connected weighted CR manifold (M, θ, f) of real dimension $2m+1$. Here $\Delta_{b,p}$ is the weighted sub-p-Laplacian with respect to θ, $q > \max\{p-1, 1\}$ is a given parameter, $V = V(x,t) > 0$ a.e. in $M \times (0,\infty)$, $V \in L^1_{loc}(M \times [0,\infty)$, $u_0 \geq 0$ a.e. on M and $u_0 \in L^1_{loc}(M)$. We show the the nonexistence of nonnagative weak solution of (1).

(二). 文獻探討及研究方法:

This class problems has a very long history. In Euclidean space, it starts in the seminal works by Gidas [1] and Gidas-Spruck [2]. One can find the series paper by Mitidieri, Pohozev, Pookhozhaev, Pucci, Serrin and H. Zou, etc. In the case of a complete Riemannian manifold, the results have a more recent history. In particular, we are interesting in the works of Grigor'yan-Kondratiev [3] and Grigor'yan-Sun [4] which used the approach originates form the work of Kurta [6], and the papers by Grigor'yan, Sun, Mastrolia, Monticelli, Punzo and Zhang (see [5] [12] [13] [10] [11] [14] [15] [16]).

(三). 結果與討論:

Let $B(x,r)$ be the geodesic ball in M of radius r centered at x, i.e.

$$B(x,r) = \{y \in M \mid d(x,y) < r\}$$

where $d(x,y)$ is the Carnot-Carathéodory distance with respect to θ. The volume form $e^{-f}\theta \wedge (d\theta)^n$ of (M,θ) is denoted by dV_θ. Then the volume of $B(x,r)$ is given by

$$Vol(B(x,r)) = \int_{B(x,r)} dV_\theta.$$

(四). 參考文獻:

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108年度專題研究計畫成果彙整表

計畫主持人：陳瑞堂	**計畫編號：**108-2115-M-003-010-
計畫名稱：光滑度量測度空間上的拉普拉斯算子	

成果項目			量化	單位	質化 （說明：各成果項目請附佐證資料或細項說明，如期刊名稱、年份、卷期、起訖頁數、證號...等）
國內	學術性論文	期刊論文	0	篇	
		研討會論文	0		
		專書	0	本	
		專書論文	0	章	
		技術報告	0	篇	
		其他	0	篇	
國外	學術性論文	期刊論文	0	篇	
		研討會論文	0		
		專書	0	本	
		專書論文	0	章	
		技術報告	0	篇	
		其他	0	篇	
參與計畫人力	本國籍	大專生	3	人次	3人
		碩士生	0		
		博士生	0		
		博士級研究人員	0		
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	非本國籍	大專生	0		
		碩士生	0		
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		專任人員	0		
其他成果 （無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）					